

AD-A133 999

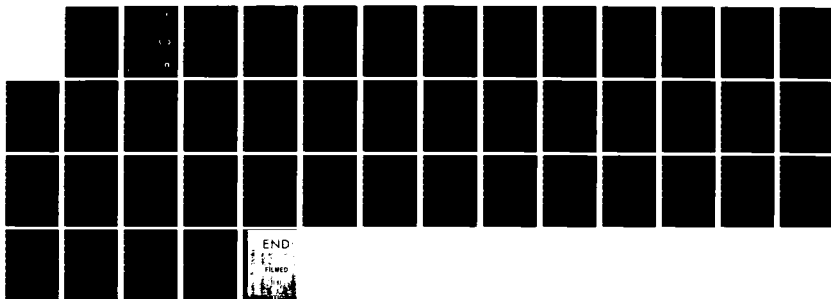
TIME LAPSE SIMULATION OF INTERRELATED WEATHER
CONDITIONS(U) AIR FORCE GEOPHYSICS LAB HANSCOM AFB MA
I I GRINGORTEN 20 MAY 83 AFGL-TR-83-0141

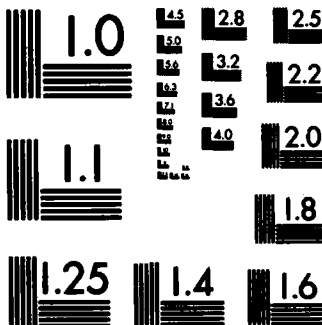
1/1

UNCLASSIFIED

F/G 4/2

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A133 999

AFGL-TR-83-0141
ENVIRONMENTAL RESEARCH PAPERS, NO. 839



12

Time Lapse Simulation of Interrelated Weather Conditions

IRVING I. GRINGORTEN

20 May 1983

DTIC
ELECTE
OCT 25 1983
S B D

Approved for public release; distribution unlimited.

METEOROLOGY DIVISION PROJECT 6670
AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MASSACHUSETTS 01731

AIR FORCE SYSTEMS COMMAND, USAF




DTIC FILE COPY

83 10 24 046

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication

FOR THE COMMANDER


for DONALD D. GRANTHAM
Chief, Tropospheric Structure Branch


ROBERT A. McCLATCHEY
Director, Meteorology Division

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

If your address has changed, or if you wish to be removed from the mailing list, or if the addressee is no longer employed by your organization, please notify AFGL/DAA, Hanscom AFB, MA 01731. This will assist us in maintaining a current mailing list.

Do not return copies of this report unless contractual obligations or notices on a specific document requires that it be returned.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-83-0141	2. GOVT ACCESSION NO. AD-A133	3. RECIPIENT'S CATALOG NUMBER 999
4. TITLE (and Subtitle) TIME LAPSE SIMULATION OF INTERRELATED WEATHER CONDITIONS	5. TYPE OF REPORT & PERIOD COVERED Scientific. Interim.	
	6. PERFORMING ORG. REPORT NUMBER ERP No. 839	
7. AUTHOR(s) Irving I. Gringorten	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Geophysics Laboratory (LYT) Hanscom AFB Massachusetts 01731	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62101F 66700910	
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory (LYT) Hanscom AFB Massachusetts 01731	12. REPORT DATE 20 May 1983	
	13. NUMBER OF PAGES 46	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified	
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Ceiling and visibility Climatic probability Cloud cover Markov process Simulation Time sequence		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Formulas or algorithms have been developed for the joint occurrence of two interrelated events changing simultaneously in a Markov time sequence. The model process is particularly applicable to a changing combination of ceiling and visibility at one station, or to the changing combination of cloud cover at two adjacent stations. A brief examination of the time sequence of three simultaneous events reveals a rapidly increasing complication of solution, making an alternative study of areal coverage of weather conditions more acceptable when more than two variables are involved.		

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Preface

→ This paper has been prompted by the need, in war games, for a stochastic model of the sequence of weather conditions as they might impinge on air/ground combat operations. Such a time sequence of weather, including ceiling, visibility, and cloud cover, for one or more stations, has been perceived as inadequately modeled, so far.

The present work is based on previously developed sequences in the Ornstein-Uhlenbeck process. Joint sequences call for a modification, but still basically comprise a Markov process. → (cont on p 1)

The effort described in this paper has been both encouraged and critically reviewed by Branch Chief, Donald D. Grantham, Tropospheric Structure Branch, Meteorology Division. The writer is grateful to his Branch peers for helpful suggestions. The author also wishes to express his appreciation to Mrs. Helen Connell for her cooperation in typing several drafts of the text and tables for this report.



Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

Contents

1. INTRODUCTION	7
2. EQUIVALENT NORMAL DEVIATE (END)	12
3. THE ORNSTEIN-UHLENBECK (O-U) PROCESS	15
4. TWO END'S IN A JOINT MARKOV PROCESS	19
5. ESTIMATING CORRELATION COEFFICIENTS BETWEEN CEILING AND VISIBILITY	22
6. TWO JOINT WEATHER SEQUENCES	24
7. MULTIPLE JOINT EVENTS IN A MARKOV PROCESS	26
8. SAMPLE SEQUENCES OF TWO INTERRELATED WEATHER CONDITIONS	30
8.1 A 48-h Sequence of Ceiling and Visibility	30
8.2 Simulation of a Two-Station 48-h Sequence of Sky Covers	33
9. DISCUSSION AND CONCLUSIONS	35
REFERENCES	39
APPENDIX A: PROCEDURE TO GENERATE A SIMULATED SEQUENCE OF END'S IN AN O-U PROCESS	41
APPENDIX B: PROCEDURE TO GENERATE A SIMULATED SEQUENCE OF TWO INTERRELATED END'S (y_1, y_2) IN A MARKOV PROCESS	43
APPENDIX C: PROCEDURE TO GENERATE A SIMULATED SEQUENCE OF TWO WEATHER ELEMENTS (x_1, x_2) IN A MARKOV PROCESS	45

Illustrations

1. The Hourly Sequence of Ceiling Height and Visibility at Boston, Mass., From 1900 EST, 21 Dec 1982 to 1900 EST, 23 Dec 1982	10
2. The Hourly Sequence of Ceiling Height at Boston, Mass., From 1900 EST, 21 Dec 1982 to 1900 EST, 23 Dec 1982	10
3. The Hourly Sequence of Visibility at Boston, Mass., From 1900 EST, 21 Dec 1982 to 1900 EST, 23 Dec 1982	11
4. The Cumulative Climatic Frequency of Ceiling Height at Bedford, Mass., in the Month of January, 12-14 EST	13
5. The Cumulative Climatic Frequency of Visibility at Bedford, Mass., in the Month of January, 12-14 EST	14
6a. Simulation of a 48-h Sequence of END's in an O-U Stochastic Process, With a 20-h Relaxation Time	17
6b. Simulation of a 48-h Sequence of END's in an O-U Process, With a 5-h Relaxation Time	17
6c. Simulation of a 48-h Sequence of END's in an O-U Process, With a 50-h Relaxation Time	18
6d. Simulation of a 48-h Sequence of END's in an O-U Process, With a 0.1-h Relaxation Time	18
7a. Simulation of END's of Ceiling Heights in a 48-h Sequence, When There is Zero Correlation With Visibility ($r_{CV} = 0$)	31
7b. Simulation of Visibility in a 48-h Sequence, When There is Zero Correlation With Ceiling ($r_{CV} = 0$)	31
7c. Simulation of Ceiling (Solid Curve) and Visibility (Broken Curve) When the Intercorrelation is $r_{CV} = 0.4$ Without Lag, $\rho_{CV} = 0.34$, $\rho_{VC} = 0.35$ With 1-h Lag	31
8. Simulation of a 48-h Sequence of Ceiling (Solid Curves) and Visibility (Dashed Curves), When Autocorrelations Are 0.95 and 0.92, Intercorrelations as Shown	32

Illustrations

9. Similar to Figure 8 With Stronger Intercorrelations	34
10. Similar to Figure 8 With Average Intercorrelations	35
11. Stochastic Simulation of a 48-h Sequence of Sky Cover, When Autocorrelations Are 0.95, $r_{12} = 0.95$, $\rho_{12} = \rho_{21} = 0.9025$	36
12. A Second Stochastic Sky-Cover Simulation	36

Tables

1. Estimates of the Joint Cumulative Probability of Ceiling, ($P \geq C$), and Visibility, ($P \geq V$), at Bedford, Mass., at Midnight	22
2. Bivariate Normal Distribution Estimates of (r_{CV}), the Tetrachoric Correlation (Zero Time Lag) Between Ceiling and Visibility, at Several END Levels, for Bedford, Mass., Midnight Values	23
3. Non-Lag Tetrachoric Correlation Coefficients Between the END's of Ceiling (C) and Visibility (V) at Bedford, Mass., in January, Derived From RUSSWO Tables	24
4. Estimates of Hour-to-Hour Autocorrelations of END's for Ceilings and Visibilities at Minneapolis, Minn., in Four Mid-Season Months	37

Time Lapse Simulation of Interrelated Weather Conditions

1. INTRODUCTION

Climatic records give the frequencies of weather elements such as ceiling and visibility, showing the cyclic variations with time of day and season of the year. Weather records of hourly observations provide sequences of events or of temporal changes of ceiling and visibility. However, there is a need for simulation of these events as a stochastic process that takes into account such attributes of the weather as the association of the visibility with the ceiling and the degree of persistence of both elements from hour to hour.

Figure 1 is a graphical record of actual Boston ceiling and visibility from midnight (Z), 22 Dec 1982 to midnight (Z) 48 h later. The first day was cloudless and clear, with both ceiling and visibility "unlimited." On the second day clouds moved in, the ceiling lowered, and visibility decreased.

Figure 2 shows the same information for the ceiling on normal probability paper. There are several advantages in plotting the sequence of cloud ceilings on Figure 2 rather than Figure 1. The dashed lines in Figure 2 show the climatic cumulative frequencies throughout the day of ceilings from 500 ft to above 30,000 ft. This graph provides the additional information that the low ceilings of 500 ft or less have an a priori probability of 2 to 3 percent in December, which is roughly for 20 h during the month. Likewise, Figure 3 shows the climatic frequencies for

(Received for publication 19 May 1983)

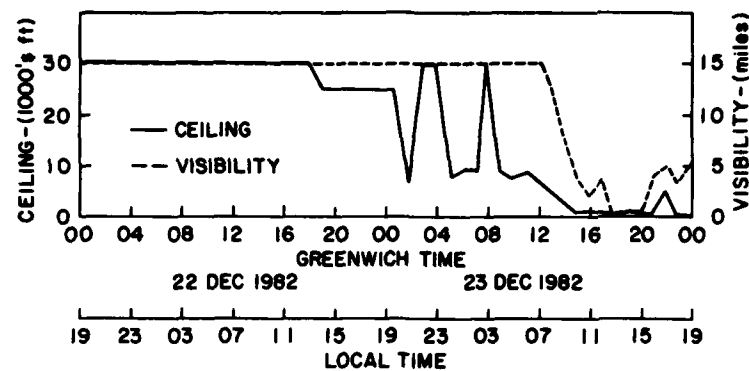


Figure 1. The Hourly Sequence of Ceiling Height and Visibility at Boston, Mass., From 1900 EST, 21 Dec 1982 to 1900 EST, 23 Dec 1982

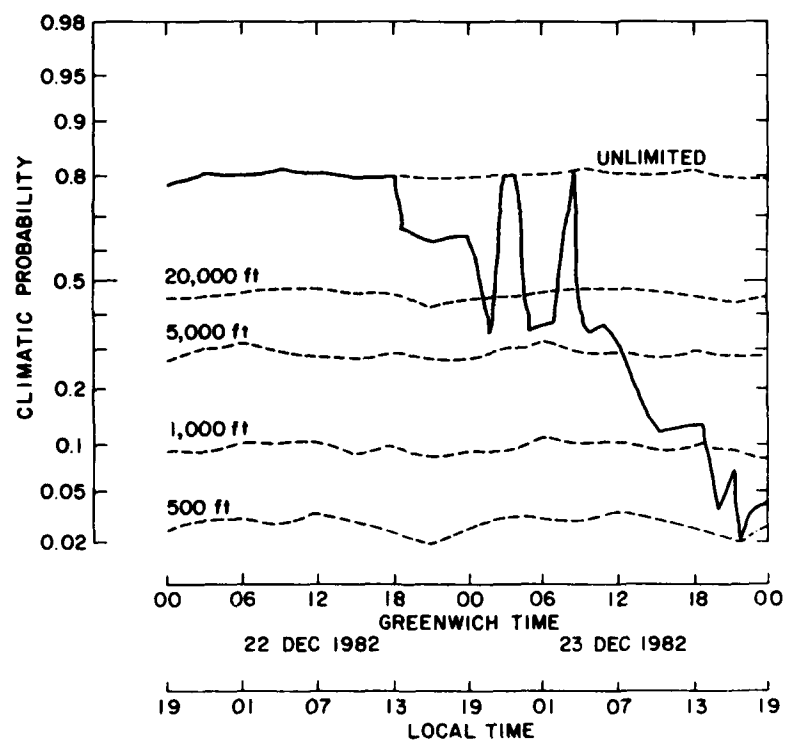


Figure 2. The Hourly Sequence of Ceiling Height at Boston, Mass., From 1900 EST, 21 Dec 1982 to 1900 EST, 23 Dec 1982. Dashed lines give the climatic frequencies of ceiling as a function of time of day

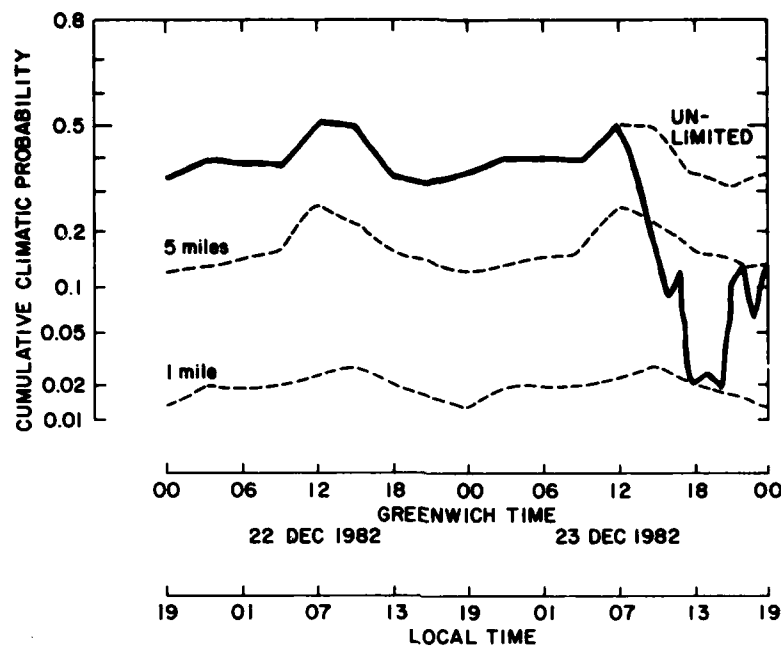


Figure 3. The Hourly Sequence of Visibility at Boston, Mass., From 1900 EST, 21 Dec 1982 to 1900 EST, 23 Dec 1982. Thin dashed lines give the climatic frequencies of visibility as a function of time of day

visibility, in comparison with the actual sequence during the 2-day period. As may be seen in subsequent examples in this paper, an improvement or a deterioration in the ceiling and visibility could be a function of the time of day.

The kind of record just described is not readily available. Even if it were, using it in tests to provide a realistic sequence of events would still be difficult. During a period as long as 50 years, many meteorological situations arise, yet they do not exhaust all the possibilities.

This report presents a stochastic procedure for simulating the changes over time in ceiling and visibility that are characteristic of a real climate. It is assumed that the changes in ceiling and visibility are a Markov process (as described in Section 3). There is a significant correlation between present ceiling and present visibility. When there is a time lag between the observations of ceiling and visibility, the correlation coefficient is reduced. The effect is explored in this report.

2. EQUIVALENT NORMAL DEVIATES (END)

Many weather elements, such as ceiling and visibility, have distributions that are difficult to treat statistically. Ceiling frequencies are usually given in terms of categories of height above the ground, and there is usually a substantial probability of occurrence of "unlimited" ceiling; that is, clear sky or scattered clouds. Modeling, however, has been reasonably successful. Bean et al¹ have used Burr curves for the cumulative distribution of ceiling heights in the form

$$F(x) = 1 - \left[1 + \left(\frac{x}{c} \right)^a \right]^{-b} \quad a, b, c > 0 \quad (1)$$

where x is ceiling height, in feet or meters, $F(x)$ is the cumulative probability of ceiling equal to, or less than, x , and a , b , and c are model parameters estimated to provide the best least squares fit. For Bedford, Mass., for example, in January, from 12 to 14 LST, the values, as given by Bean, are

$$a = 1.16779$$

$$b = 0.192682$$

$$c = 1000$$

which give the solid curve plotted in Figure 4. The RUSSWO* information on the cumulative frequencies of ceiling height are shown (by x's) on Figure 4. The model fits well, but requires some caution in application, because unless it is the right tool for the problem, the answer it gives might be misleading. For example, the ceiling is unlimited with an observed frequency of 0.445. This is comparable to the formula estimate of probability of ceiling above 32,000 ft. Ceiling less than 100 ft should be categorized with a probability of 0.005.

Visibility frequencies, likewise, have been fitted by an idealized model. Somerville et al² use the Weibull distribution to give the cumulative probability of visibility (x) as

$$F(x) = 1 - \exp(-\alpha x^\beta) \quad (2)$$

*RUSSWO stands for "Revised Uniform Summaries of Surface Weather Observations," published by USAF Environmental Technical Application Center for several hundred stations around the world.

1. Bean, S.J., Somerville, P.N., and Heuser, M. (1979) Some Models for Ceiling, Scientific Report No. 7, Contract F19628-77-C-0080, AD A078033.
2. Somerville, P.N., Bean, S.J., and Falls, F. (1979) Some Models for Visibility, Scientific Report No. 3, Contract F19628-77-C-0080, AD A075490.

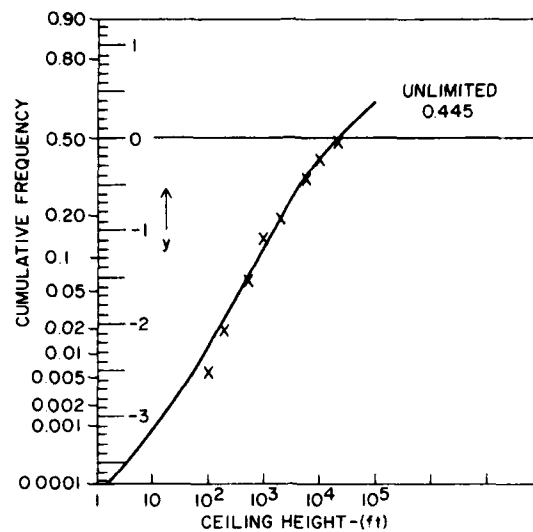


Figure 4. The Cumulative Climatic Frequency of Ceiling Height at Bedford, Mass., in the Month of January, 12-14 EST. The X's mark the frequencies as given in RUSSWO tables. The solid line is the "best" model fit by a Burr curve

where α and β are parameters. For Bedford, Mass., in January, from 12 to 14 LST, the values are:

$$\alpha = 0.06906$$

$$\beta = 0.8186$$

which give the curve as plotted in Figure 5. The RUSSWO data again reveal that the model fits well, but visibility less than 1/4 mile should be categorized with a probability of 2 percent. Visibility greater than 10 miles should be categorized with a probability of 62 percent.

Any probability $F(x)$ as given in Eq. (1) or (2), corresponds to an equivalent normal deviate (END), symbolized by $y(0, 1)$. The latter is a variable with Gaussian distribution, mean value of 0, and standard deviation 1.0. Symbolically,

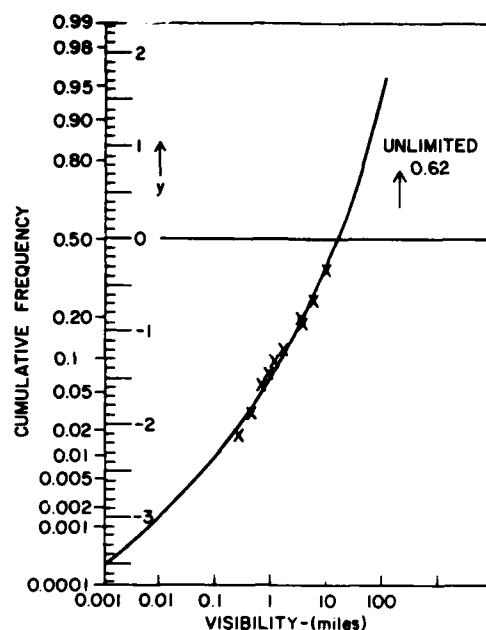


Figure 5. The Cumulative Climatic Frequency of Visibility at Bedford, Mass., in the Month of January, 12-14 EST. The X's mark the RUSSWO frequencies. The solid curve is the "best" model fit by a Weibull distribution

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y \exp - \frac{\gamma^2}{2} d\gamma \quad (3)$$

which thus defines the END (y). The latter is implicitly found for each x.

To a highly satisfactory approximation,³ (error < 0.003)

$$y = k \left[t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2} \right]$$

where

$$a_0 = 2.30753$$

$$a_1 = 0.27061$$

3. NBS (1964) Handbook of Mathematical Functions, Applied Mathematics Series, No. 55, Government Printing Office, Washington, D.C.

$$b_1 = 0.99229$$

$$b_2 = 0.04481$$

and

$$k = 1, t = \sqrt{\ln \frac{1}{p^2}} \quad \text{when } p = F(x) \leq 1/2$$

or

(4)

$$k = -1, t = \sqrt{\ln \frac{1}{(1-p)^2}} \quad \text{when } p = F(x) > 1/2$$

A scale of END, (y) appears alongside the scale of F(x) in both Figures 4 and 5.

In this report, ceiling height and visibility are modeled in terms of ENDs because they have varying averages and medians throughout the day, and will have generally uneven distributions. But the END of the ceiling or visibility will have a symmetrical Gaussian distribution with a constant mean or median of zero. In Figure 3 it can be seen how, for example, visibility less than 5 miles has varying probability throughout the day. The END varies directly with this probability. Correlations in this paper are found between the END's of ceilings at *differing* times of day, or between the END's of ceiling and visibility, with or without time lag.

3. THE ORNSTEIN-UHLENBECK (O-U) PROCESS

A Markov process is defined as "a stochastic process such that the conditional probability distribution for the state at any future instant, given the present state, is unaffected by any additional knowledge of the past history of the system".⁴ The Ornstein-Uhlenbeck (O-U) process is one kind of Markov process in which the value at a future instant $t + \delta t$, of a normally distributed variable y is linearly related to the value at the present instant t , and the correlation coefficient between present and future values decays exponentially with the time interval δt between them. Mathematically, this may be stated as follows:

4. Kendall, M.G., and Buckland, W.R. (1971) A Dictionary of Statistical Terms, Hafner Publishing Co., New York.

$$y(t + \delta t) = \rho \cdot y(t) + \sqrt{1 - \rho^2} \cdot \eta(t + \delta t) \quad (5)$$

where the correlation coefficient ρ is given by

$$\rho = \exp(-\delta t / \tau) \quad (6)$$

where τ is a parameter with the dimension of time, named the Relaxation Time, by Keilson and Ross,⁵ and described by Gringorten.⁶ The random number η is normally distributed and is the random contribution to the process of change.

A time sequence of y's at regular intervals (δt), generated by the model of Eq. (5), will simulate the conditions of a weather element in terms of its END. A step-by-step program to accomplish this is given in Appendix A.

Figure 6a was drawn for a relaxation time τ of 20 h, which corresponds to a realistic hour-to-hour correlation coefficient of 0.95. Figure 6a presents the simulation of sky cover for a partly cloudy to overcast 48-h period. The dashed lines show climatic frequencies of clear, scattered, broken and overcast throughout a day in August at Bedford, Mass., deliberately chosen because of a large diurnal effect. Figure 6b illustrates a sequence in which serial correlation is weaker (0.82) with $\tau = 5$ h. Still, with this much persistence an overcast might remain, with few breaks, for 20 h. There is a clearing at the most likely time of the 24-h period. Figure 6c illustrates a sequence in which serial correlation is stronger (0.98) with $\tau = 50$ h. Finally, Figure 6d illustrates a sequence in which hour-to-hour correlation is reduced virtually to zero, with $\tau = 0.1$ h, producing rapid changes and short periods of all sky-cover conditions.

5. Keilson, J., and Ross, H. F. (1979) Gaussian Markov Related Variates for Meteorological Planning, Final Report, Contract F19628-78-C-0158, AD A081382.

6. Gringorten, I. I. (1982) The Keilson-Ross Procedure for Estimating Climatic Probabilities of Duration of Weather Conditions, AFGL-TR-82-0116, AD A119860.

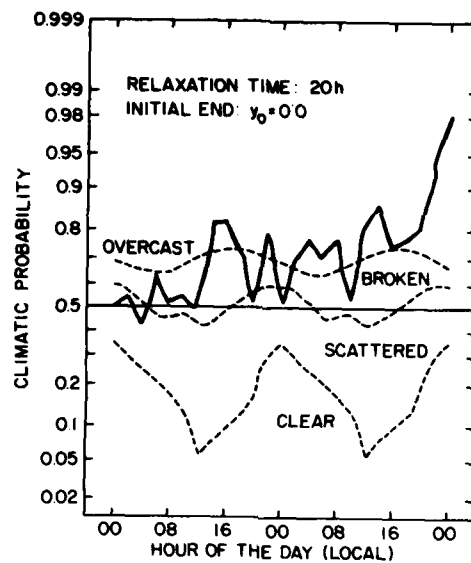


Figure 6a. Simulation of a 48-h Sequence of END's in an O-U Stochastic Process, With a 20-h Relaxation Time. The dashed lines show climatic frequencies of clear, scattered, broken, and overcast throughout a day in August at Bedford, Mass.

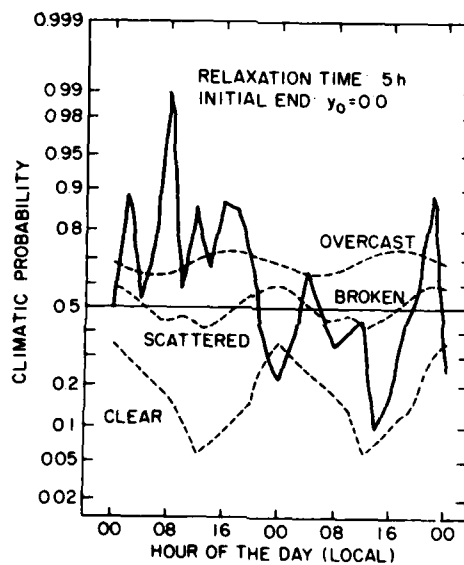


Figure 6b. Simulation of a 48-h Sequence of END's in an O-U Process, With a 5-h Relaxation Time

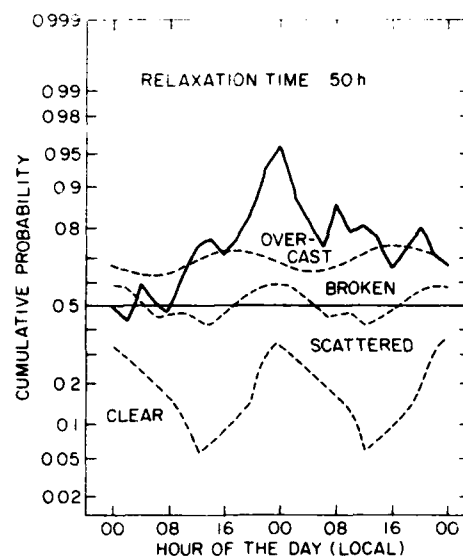


Figure 6c. Simulation of a 48-h Sequence of END's in an O-U Process, With a 50-h Relaxation Time

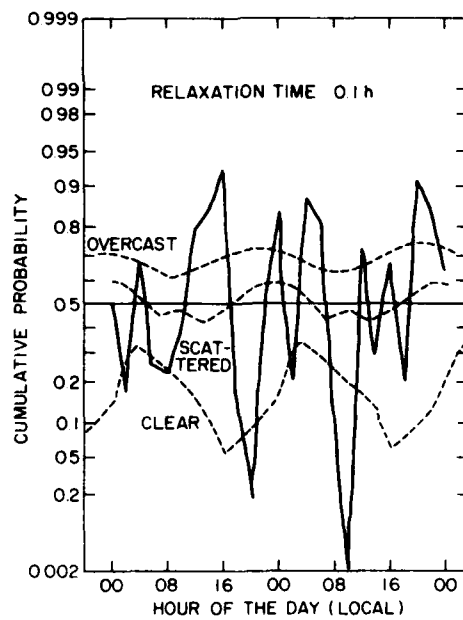


Figure 6d. Simulation of a 48-h Sequence of END's in an O-U Process, With a 0.1-h Relaxation Time

4. TWO END'S IN A JOINT MARKOV PROCESS

The above procedure gives a time lapse simulation of a single weather element. The next task is to provide the algorithms for time lapse simulation of two elements that are interrelated. Two such weather conditions (X_i, X_j) may be the two sky covers at two neighboring stations, or they may be ceiling and visibility at one station. Whatever they be, in this paper they are represented by their END's (y_i, y_j), each of whose means is zero, standard deviation 1.0, and whose probability distribution is Gaussian.

The two END's, $y_i(t)$ and $y_j(t)$ corresponding to the two events (X_i, X_j) at time (t) are subject to change in the time interval (δt) in a Markov process, given as follows:

$$\begin{aligned} y_i(t + \delta t) &= a_{ii} \cdot y_i(t) + a_{ij} \cdot y_j(t) + b_i \cdot \eta_i(t + \delta t) \\ y_j(t + \delta t) &= a_{ji} \cdot y_i(t) + a_{jj} \cdot y_j(t) + b_j \cdot \eta_j(t + \delta t) \end{aligned} \quad (7)$$

where η_i, η_j are normally distributed and random, except for their interrelationship. The a 's are partial regression coefficients and the b 's are of such magnitude that the normality of y_i, y_j is preserved. The a 's and b 's need to be determined in terms of correlation coefficients, which are derivable from the climatology of the station or stations.

Because of the normality of the END's the correlation coefficients, by definition, are:

$$r_{ij} = E[y_i(t) \cdot y_j(t)] = E[y_i(t + \delta t) \cdot y_j(t + \delta t)]$$

where the symbol $E[]$ denotes the expected value of the quantity in brackets, and r_{ij} is the correlation coefficient between the elements without time lag. For time lag (δt):

$$\begin{aligned} \rho_{ii} &= E[y_i(t) \cdot y_i(t + \delta t)] \\ \rho_{ij} &= E[y_i(t) \cdot y_j(t + \delta t)] \\ \rho_{ji} &= E[y_j(t) \cdot y_i(t + \delta t)] \\ \rho_{jj} &= E[y_j(t) \cdot y_j(t + \delta t)] \end{aligned} \quad (9)$$

Also, the variances are:

$$E y_i^2(t) = E y_j^2(t) = E \eta_i^2 = E \eta_j^2 = 1 .$$

The mean and variance of each END do not vary with time.

From Eqs. (7) and (8), after squaring both sides of each equation:

$$\begin{aligned} 1 &= a_{ii}^2 + a_{ij}^2 + 2a_{ii} a_{ij} r_{ij} + b_i^2 \\ 1 &= a_{ji}^2 + a_{jj}^2 + 2a_{ji} a_{jj} r_{ij} + b_j^2 \end{aligned} \quad (10)$$

The derivation of these equations takes advantage of the fact that η_i and η_j are obtained independently of y_i and y_j .

Again, from Eqs. (7) and (9):

$$\begin{aligned} \rho_{ii} &= a_{ii} + a_{ij} r_{ij} \\ \rho_{ij} &= a_{ji} + a_{jj} r_{ij} \\ \rho_{ji} &= a_{ii} r_{ij} + a_{ij} \\ \rho_{jj} &= a_{ji} r_{ij} + a_{jj} \end{aligned} \quad (11)$$

Solving for the a's:

$$\begin{aligned} a_{ii} &= (\rho_{ii} - \rho_{ji} r_{ij}) / (1 - r_{ij}^2) \\ a_{ij} &= (\rho_{ji} - \rho_{ii} r_{ij}) / (1 - r_{ij}^2) \\ a_{ji} &= (\rho_{ij} - \rho_{jj} r_{ij}) / (1 - r_{ij}^2) \\ a_{jj} &= (\rho_{jj} - \rho_{ij} r_{ij}) / (1 - r_{ij}^2) \end{aligned} \quad (12)$$

From Eq. (10):

$$\begin{aligned} b_i &= \sqrt{1 - \{a_{ii}^2 + a_{ij}^2 + 2 a_{ii} a_{ij} r_{ij}\}} \\ b_j &= \sqrt{1 - \{a_{ji}^2 + a_{jj}^2 + 2 a_{ji} a_{jj} r_{ij}\}} \end{aligned} \quad (13)$$

There remains the question of the interrelation of the stochastically produced values, $\eta_i(t + \delta t)$ and $\eta_j(t + \delta t)$. Symbolizing the correlation coefficient between them as h , we find from Eq. (7):

$$b_i b_j \cdot h = E \{ y_i(t + \delta t) - a_{ii} \cdot y_i(t) - a_{ij} \cdot y_j(t) \} \cdot \\ \{ y_j(t + \delta t) - a_{ji} \cdot y_i(t) - a_{jj} \cdot y_j(t) \}$$

Hence,

$$h = \{ r_{ij}(1 - a_{ii} a_{jj} - a_{ij} a_{ji}) - (a_{ii} a_{ji} + a_{ij} a_{jj}) \} / b_i b_j \quad (14)$$

If two random, normal numbers $\eta_i(t + \delta t)$, $\eta_j(t + \delta t)$ are selected, then:

$$\eta_j(t + \delta t) = h \cdot \eta_i(t + \delta t) + \sqrt{1 - h^2} \cdot \eta_j^!(t + \delta t) \quad (15)$$

Thus, all the values are obtainable for the solution of Eq. (7) if the correlation coefficients are provided. By substitution of $y_i(t + \delta t)$, $y_j(t + \delta t)$ for $y_i(t)$, $y_j(t)$ the equations can be solved for the next pair of y_i and y_j . By such iteration, the simulation of a joint sequence of values of y_i , y_j is obtainable. The computer programming of this process is given in Appendix B.

A variation of the above solution was obtained by Maj. R. C. Whiton⁷ at USAF/ETAC, Scott AFB, Ill. Whereas Eq. (7) presupposes that the later value of each of the ceiling and visibility is dependent on both the previous ceiling and the previous visibility, Whiton's equation for the later ceiling assumed its dependence on the current ceiling, but not on the current visibility; likewise for the later visibility. Thus the process of change in either ceiling or visibility satisfies the definition of an O-U process. This is equivalent to setting the lag correlation equal to the product:

$$\rho_{ij} = r_{ij} \cdot \rho_{jj} \\ \rho_{ji} = r_{ij} \cdot \rho_{ii} \quad (16)$$

Whiton's simplification will often prove effective. However, for generality it might be better to avoid this assumption and let the climatic data provide numbers for ρ_{ij} , ρ_{ji} . Some sources of the correlation coefficients are given and discussed in the following sections.

7. Whiton, R.C. (1982) Environmental Simulation Modeling, AWS/ETAC, Scott AFB, Ill.

5. ESTIMATING CORRELATION COEFFICIENTS BETWEEN CEILING AND VISIBILITY

A measure of the intercorrelation, without time lag, between ceiling and visibility is obtainable from a RUSSWO. Table 1, for example, presents the joint probabilities ($P(\geq C, \geq V)$), as interpolated from the RUSSWO tables for Bedford, MA, midnight, in January, April, July, and October. The probabilities, as selected, are the same, in pairs, for ceiling and for visibility. The corresponding END's are also shown in Table 1. "Tables of the Bivariate Normal Distribution"⁸ (NBS, 1957) make it possible to find, by interpolation, the intercorrelation coefficient (ρ), given the probabilities $P(\geq C)$, $P(\geq V)$, or their END's, together with their joint probability $P(\geq C, \geq V)$. Correlation that is estimated from these probabilities is known as tetrachoric correlation.⁹

Table 1. Estimates of the Joint Cumulative Probability of Ceiling, $P(\geq C)$, and Visibility, $P(\geq V)$, at Bedford, Mass., at Midnight. The conditions of ceiling and visibility are those corresponding to their percentiles: 60, 70, 80, and 90 percent

$P(\geq C)$ = $P(\geq V)$	END	Month			
		January	April	July	October
0.6	0.25	0.47	0.48	0.43	0.42
0.7	0.53	0.60	0.60	0.56	0.54
0.8	0.84	0.75	0.74	0.70	0.73
0.9	1.28	0.88	0.86	0.85	0.86

A computerized solution of the intercorrelation coefficient may be found to satisfy the relation upon which the NBS tables are based:

$$P(\geq x, \geq y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_x^\infty \int_y^\infty e^{-\frac{\xi^2 + \eta^2 - 2\rho\xi\eta}{2(1-\rho^2)}} \cdot d\xi \cdot d\eta \quad (17)$$

Programming such an equation on a desk-top computer is difficult and provides only approximate solutions. The computer operation is slow because it requires trial-and-error iterations.

8. NBS (1957) Tables of the Bivariate Normal Distribution Function, Applied Mathematics Series, No. 50, Government Printing Office, Washington, D.C.
9. Brooks, C.E.P., and Carruthers, N. (1953) Handbook of Statistical Methods in Meteorology, Her Majesty's Stationery Office, London.

A reasonable estimate of the tetrachoric correlation coefficient has been presented,⁹ as follows:

$$r_{xy} = \sin \left[\frac{\pi}{2} \frac{\sqrt{ad} - \sqrt{bc}}{\sqrt{ad} + \sqrt{bc}} \right] \quad (18)$$

where

$$\begin{aligned} a &= P(\geq X, \geq Y) \\ b &= P(\geq X) - a \\ c &= P(\geq Y) - a \\ d &= 1 - a - b - c \end{aligned}$$

The results of using this formula are shown in the parentheses (Table 2), for easy comparison with the results of applying a computerized approximation of the bivariate normal distribution. Since there is little or no significant difference in the results, Eq. (18) is favored because it is a simpler method for finding the correlation coefficient (r_{xy}).

Table 2. Bivariate Normal Distribution Estimates of (r_{CV}), the Tetrachoric Correlation (Zero Time Lag) Between Ceiling and Visibility, at Several END Levels, for Bedford, Mass. Midnight Values. Figures in parentheses were obtained by using the model of Eq. (18), from Brooks and Carruthers⁹

P($\geq C$) = P($\geq V$)	END	Month			
		January	April	July	October
0.6	0.25	0.66 (0.67)	0.72 (0.72)	0.47 (0.45)	0.40 (0.39)
0.7	0.53	0.78 (0.76)	0.78 (0.76)	0.50 (0.54)	0.40 (0.40)
0.8	0.84	0.91 (0.92)	0.84 (0.88)	0.59 (0.65)	0.84 (0.84)
0.9	1.28	0.93 (0.98)	0.90 (0.89)	0.80 (0.82)	0.85 (0.89)

Using the formula of Eq. (18), the tetrachoric correlation coefficient was found for the RUSSWO frequencies of ceiling (C) and visibility (V) and joint frequencies at Bedford, Mass., in January, at midnight and at 12 noon (Table 3). There are no entries for ceiling less than 2,000 ft or for visibility less than 2 miles because the samples were too small to give meaningful figures.

Table 3. Non-Lag Tetrachoric Correlation Coefficients Between the END's of Ceiling (C) and Visibility (V) at Bedford, Mass., in January, Derived From RUSSWO Tables. In each box the upper figure is the midnight value, the lower figure the noontime value

C	P(\geq C)	V	≥ 10	≥ 6	≥ 4	≥ 2
		P(\geq V)	0.592 0.623	0.735 0.746	0.831 0.818	0.887 0.884
$\geq 30,000$	0.507		0.64	0.72	0.81	0.82
	0.445		0.78	0.89	0.94	1.00
$\geq 10,000$	0.597		0.70	0.76	0.84	0.84
	0.599		0.80	0.87	0.94	1.00
$\geq 5,000$	0.684		0.75	0.82	0.86	0.88
	0.675		0.80	0.86	0.92	0.98
$\geq 2,000$	0.805		0.92	0.91	0.92	0.94
	0.805		0.95	0.93	0.94	0.97

Unfortunately, the tetrachoric correlation coefficient shows considerable dependence on the specific values of ceiling and visibility, or alternatively on the probabilities of the events. It can be expected that low ceilings and visibilities will be highly correlated, both having low probabilities of occurrence. We next examine this effect further.

6. TWO JOINT WEATHER SEQUENCES

By the procedure of Section 4, we are able to generate pairs of values of END's, for example, one for ceiling and one for visibility. For practical purposes these should be transformed into units of ceiling and visibility at one station, or into the units of ceiling at two neighboring stations.

An END (y) can be transformed into its probability,³ which is also the probability of the original variable (X). Thus, using the notation $P(\leq X)$ for the cumulative probability of X , we are given (with 3-decimal accuracy):

$$P(\leq X) = P(\leq y) = 1 + m [2(1 + c_1|y| + c_2y^2 + c_3|y^3| + c_4y^4)^4]^{-1} \quad (19)$$

where

$$c_1 = 0.196854$$

$$c_2 = 0.115194$$

$$c_3 = 0.000344$$

$$c_4 = 0.019527$$

$$l = 1, m = -1 \text{ for } y \geq 0$$

$$l = 0, m = 1 \text{ for } y < 0$$

If the Burr curve¹ is accepted as the model for the distribution of ceiling height (C) then C is given by

$$C = c[(1 - p)^{-1/b} - 1]^{1/a} \quad (20)$$

where $p = P(\leq C)$; a, b, and c are parameters. Bean et al¹ give tables of values for these three parameters to yield answers for ceiling heights in feet.

If the Weibull distribution² is accepted as the model for visibility (V) then V is given by

$$V = \left[\frac{1}{\alpha} \ln(1 - p) \right]^{1/\beta} \quad (21)$$

where $p = P(\leq V)$, and α and β are parameters. Somerville et al² give tables of values for these two parameters to yield visibility in miles.

The procedure for generating a stochastic sequence of joint events is linked to the previously outlined procedure (Section 4) in the following order:

At each step in the sequence, pairs of random normal numbers are selected and used with the known correlation coefficients for the determination of the END's [Eq. (7)]. Each END (y) is then transformed into the corresponding probability $p = P(\geq y)$ by Eq. (19). Then each p is transformed into the weather element itself, into ceiling height by Eq. (20), or into visibility by Eq. (21). The computer program is in Appendix C.

7. MULTIPLE JOINT EVENTS IN A MARKOV PROCESS

The above derivation could be generalized for n weather elements in the form:

$$y_i = \sum_j a_{ij} x_j + b_i \cdot \eta_i \quad \text{for } i = 1, n \quad (22)$$

where the y 's are the predictands and the x 's are the predictors. Where A_i is a column vector, A_i^T is a row vector of n coefficients:

$$A_i^T = (a_{i1}, a_{i2}, \dots, a_{in}) \quad (23)$$

Squaring Eq. (22) and taking expected values:

$$b_i = \sqrt{1 - A_i^T C A_i} \quad \text{for } i = 1, n \quad (24)$$

where C is the matrix of correlation coefficients (r_{ij}) between the n predictors without lag:

$$C = \begin{bmatrix} 1, & r_{12}, & \dots, & r_{1n} \\ r_{21} & & & \\ \cdot & & & \\ \cdot & & & \\ r_{n1}, & \dots, & 1 \end{bmatrix} \quad (25)$$

Eq. (22) represents n equations, each of the form

$$a_{i1} x_1 + \dots + a_{in} x_n + b_i \eta_i = y_i \quad \text{for } i = 1, n \quad (26)$$

To solve for the n coefficients (a_{i1}, \dots, a_{in}) we find n equations by multiplying Eq. (26) successively by x_1, \dots, x_n and then taking expected values. This procedure introduces the correlation coefficients (r_{ij}) between the predictors without time lag, and the correlation coefficients (ρ_{ij}) between the predictand (y_i) and each of the predictors ($x_j, j = 1, n$). Thus the n equations become

$$a_{i1} + a_{i2} r_{12} + \dots + a_{in} r_{1n} = \rho_{i1}$$

$$a_{i1} r_{21} + a_{i2} + \dots + a_{in} r_{2n} = \rho_{i2} \quad (27)$$

...

$$a_{i1} r_{n1} + a_{i2} r_{n2} + \dots + a_{in} = \rho_{in}$$

In matrix form:

$$CA_i = R_i$$

$$\text{to give } A_i = C^{-1}R_i \quad (28)$$

where R_i is the column vector of the lag correlation coefficients between the predictand (y_i) and each of the predictors (x_{i1}, \dots, x_{in}). Thus,

$$R_i = \begin{bmatrix} \rho_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \rho_{in} \end{bmatrix} \quad (29)$$

and C^{-1} is the inverse matrix of C ,

$$C^{-1} = \hat{C}/|C|, \quad (30)$$

where $|C|$ is the determinant of C , \hat{C} is the adjoint matrix of C , that is, the matrix of cofactors (p_{ij}),

$$p_{ij} = (-1)^{i+j} |M_{ij}| \quad 1 \leq i \leq n, 1 \leq j \leq n, \quad (31)$$

and M_{ij} is the submatrix of order $(n - 1)$ obtained by deleting the i th row and the j th column of C .

After the a 's are solved in terms of the correlation coefficients, and the b 's are solved by Eq. (24), there remains the problem of finding the interrelation of the random components (η_i) for use in Eq. (22), such that the a priori correlation coefficient between future values of y_i is preserved.

From Eq. (22):

$$\begin{aligned} b_i \eta_i &= y_i - a_{i1}x_1 \dots - a_{in}x_n \\ b_j \eta_j &= y_j - a_{j1}x_1 \dots - a_{jn}x_n \end{aligned} \quad (32)$$

If h_{ij} is the correlation coefficient between η_i and η_j then

$$b_i b_j h_{ij} = r_{ij} - \sum_{k=1}^n a_{ik} \rho_{jk} - \sum_{k=1}^n a_{jk} \rho_{ik} + \sum_{k,l} a_{ik} \cdot a_{jl} r_{kl} \quad (33)$$

which defines the $\binom{n}{2}$ correlation coefficients between the η 's. [$\binom{n}{2}$ represents the number of possible combinations of n things, taken two at a time.] To find values for η_j , ($j = 1, n$), for use in Eqs. (19), first choose n random, independent numbers

$$(\zeta_i, i = 1, n) \quad .$$

The relations of the η 's to the ζ 's are of the form:

$$\begin{aligned} \alpha_{11} \zeta_1 + \dots + \alpha_{1n} \zeta_n &= \eta_1 \\ \alpha_{21} \zeta_1 + \dots + \alpha_{2n} \zeta_n &= \eta_2 \\ \alpha_n \zeta_1 + \dots + \alpha_{nn} \zeta_n &= \eta_n \end{aligned} \quad (34)$$

The task, now, is to find suitable values for the α 's. There are n^2 values for the α 's but only $n + \binom{n}{2}$ equations are obtainable by setting $E\eta_i^2 = 1$, and by finding the $\binom{n}{2}$ values of h_{ij} from Eq. (33). A certain arbitrariness, therefore, is preserved in assigning values to some of the α 's.

For $n = 2$, or for only two weather elements, as seen previously, if we can set $\alpha_{11} = 1$, it follows:

$$\alpha_{12} = 0$$

$$\alpha_{21} = h_{12} \quad (35)$$

$$\alpha_{22} = \sqrt{1 - h_{12}^2}$$

For $n = 3$, from Eq. (31), six equations can be written:

$$\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{13}^2 = 1$$

$$\alpha_{21}^2 + \alpha_{22}^2 + \alpha_{23}^2 = 1$$

$$\alpha_{31}^2 + \alpha_{32}^2 + \alpha_{33}^2 = 1$$

(36)

$$\alpha_{11} \alpha_{21} + \alpha_{12} \alpha_{22} + \alpha_{13} \alpha_{23} = h_{12}$$

$$\alpha_{21} \alpha_{31} + \alpha_{22} \alpha_{32} + \alpha_{23} \alpha_{33} = h_{23}$$

$$\alpha_{31} \alpha_{11} + \alpha_{32} \alpha_{12} + \alpha_{33} \alpha_{13} = h_{31}$$

If $\alpha_{11} = 1$, then it follows that $\alpha_{12} = \alpha_{13} = 0$, $\alpha_{21} = h_{12}$, $\alpha_{31} = h_{31}$. If $\alpha_{23} = 0$, then:

$$\alpha_{22} = \sqrt{1 - h_{12}^2}$$

$$\alpha_{32} = (h_{23} - h_{12} h_{31}) / \sqrt{1 - h_{12}^2}$$

$$\alpha_{33} = \sqrt{(1 - h_{12}^2 - h_{23}^2 - h_{31}^2 + 2 h_{12} h_{23} h_{31}) / (1 - h_{12}^2)}$$

to complete values for all nine α 's.

This aspect of time lapse simulation, of simultaneous sequences of three or more interrelated weather conditions in an area, is not pursued further, in the

belief that modeling the probability of fractional cover of areas of varying size¹⁰ will provide a more descriptive mechanism for simulation of multiple (≥ 3) conditions.

8. SAMPLE SEQUENCES OF TWO INTERRELATED WEATHER CONDITIONS

8.1 A 48-h Sequence of Ceiling and Visibility

In temperate latitudes and continental regions, the hour-to-hour correlation coefficient, in January, has been found to be approximately $\rho_{CC} = 0.95$ for ceilings, $\rho_{VV} = 0.92$ for visibilities.¹¹ The correlation coefficients between ceiling and visibility without time lag have been found to vary significantly with the specific values of ceiling and visibility (Table 3). This effect on the simulation process is examined in this section. Additionally, since the lag correlation coefficients between ceiling and visibility should be smaller than the non-lag correlation coefficients, it is desirable to see what happens when these lag correlation coefficients (ρ_{CV}^1 , ρ_{VC}^1) are less than, or greater than, the values given by the Whiton limitation [Eq. (16)].

Figure 7a shows the changes in END of ceiling height for 48 consecutive hours. They were generated stochastically by use of the above equations, when the intercorrelation (r_{CV}) between ceiling and visibility was set at zero. That is, ceiling and visibility are represented as changing independently of one another. Figure 7b shows the corresponding changes of visibility. The dashed lines on Figures 7a and 7b are based on Bedford, Mass. RUSSWO data for the month of January. They indicate that, climatically speaking, ceiling is unlimited about 50 percent of the time, and visibility is unlimited approximately 60 percent of the time, varying little with time of day. As long as the END is greater than that shown by the upper dashed line, ceiling or visibility is unlimited. The imaginary sample of Figures 7a and 7b is such that the ceiling was virtually unlimited throughout the 2 days, but visibility was frequently restricted to less than 10 miles.

Figure 7c shows joint variations of ceiling and visibility when there is a relatively small non-lag correlation coefficient of 0.4 between the END's of ceiling and visibility and lag correlation coefficients of 0.34 and 0.35. They were produced by the same random numbers as were used in Figures 7a and 7b.

Figures 8(a), 8(b), and 8(c) show a stochastic 48-h sequence of Bedford, Mass., January ceiling and visibility when there was supposedly a non-lag

10. Gringorten, I.I. (1979) Probability models of weather conditions occupying a line or an area, J. Appl. Meteorol. 18(No. 8):957-977.
11. Gringorten, I.I. (1966) A stochastic model of the frequency and duration of weather events, J. Appl. Meteorol. 5(No. 5):606-624.

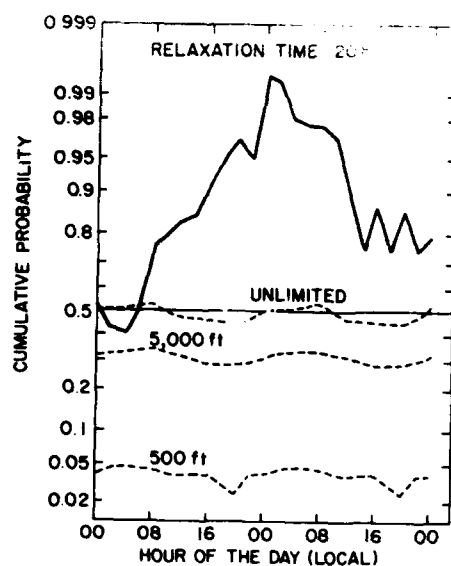


Figure 7a. Simulation of END's of Ceiling Heights in a 48-h Sequence, When There is Zero Correlation With Visibility ($r_{CV} = 0$). Dashed lines show climatic frequencies of ceiling heights at Bedford, Mass. in January. Hour-to-hour autocorrelation of ceilings is 0.95, of visibility 0.92

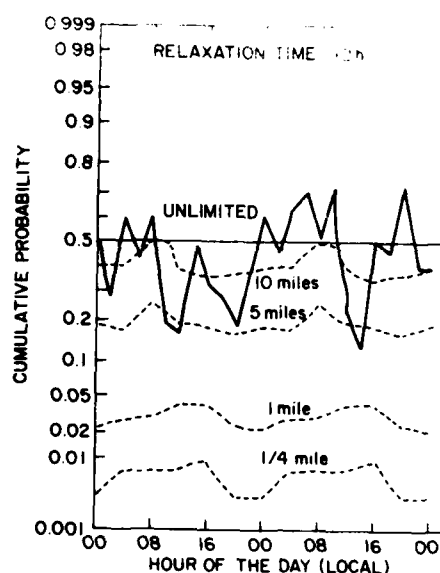


Figure 7b. Simulation of Visibility in a 48-h Sequence, When There is Zero Correlation With Ceiling ($r_{CV} = 0$). Dashed lines show climatic frequencies of visibility at Bedford, Mass. in January. Autocorrelations are as in Figure 7a

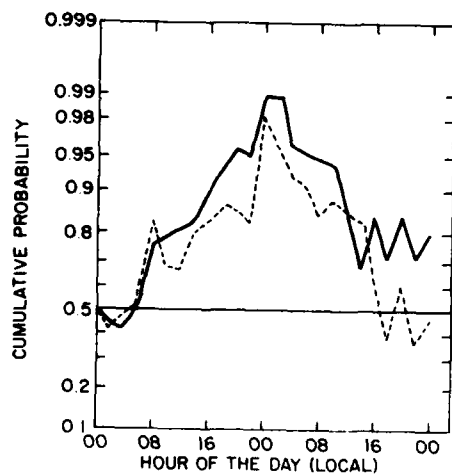


Figure 7c. Simulation of Ceiling (Solid Curve) and Visibility (Broken Curve) When the Intercorrelation is $r_{CV} = 0.4$ Without Lag, $\rho_{CV} = 0.34$, $\rho_{VC} = 0.35$ With 1-h Lag. Lines for the climatic frequencies are omitted; they would be identical to those in Figures 7a and 7b

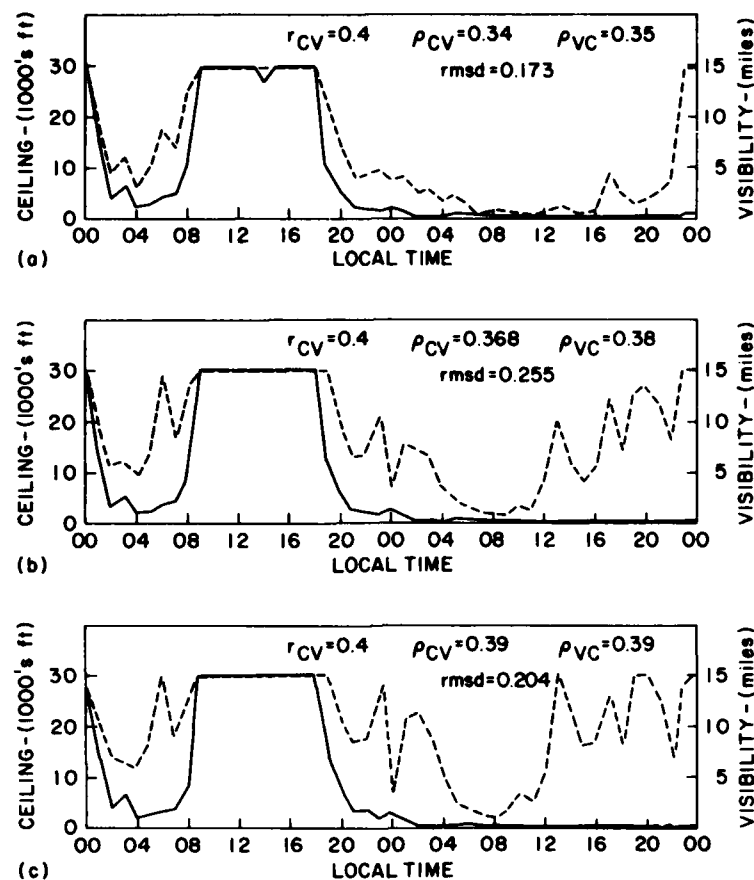


Figure 8. Simulation of a 48-h Sequence of Ceiling (Solid Curves) and Visibility (Dashed Curves), When Autocorrelations are 0.95 and 0.92, Intercorrelations as Shown. The curves are intended to simulate Bedford, Mass. January ceilings and visibilities

correlation coefficient (r_{CV}) of 0.4 between their END's. The autocorrelations were the same as in Figure 7 ($\rho_{CC} = 0.95$, $\rho_{VV} = 0.92$). The ceiling and visibility, in Figure 8 are plotted directly in feet and miles by the method of Appendix C. In Figure 8(b) the curves were generated when the lag correlation coefficients were made subject to the Whiton limitation, and given by Eq. (16), thus ($\rho_{CV} = 0.368$, $\rho_{VC} = 0.38$). In Figure 8(a) the lag correlation coefficients were made

slightly less ($\rho_{CV} = 0.34$, $\rho_{VC} = 0.35$). In Figure 8(c) the lag correlation coefficients were made slightly greater ($\rho_{CV} = \rho_{VC} = 0.39$). Ironically, the ceiling and visibility were coupled together most closely when the lag correlation coefficients were the smallest, but this result was not consistently repeated in other trials.

Figures 9(a), 9(b), and 9(c) show the 48-h sequence of Bedford, Mass., January ceiling and visibility, stochastically produced with the same random numbers as in Figures 8(a), 8(b), and 8(c). This time there was supposedly a non-lag correlation coefficient of 0.95 between the END's of ceiling and visibility. The lag correlation coefficients (ρ_{CV} , ρ_{VC}) were as low as they could be, for practical solution, in Figure 9(a). The Whiton values were used in Figure 9(b), and relatively high values in Figure 9(c). It appears that the correlation coefficient (r_{CV}) between ceiling and visibility made its greatest difference on the rare events, close to zero ceiling and/or zero visibility, when the intercorrelation is, in fact, the highest.

Faced with an array of correlation coefficients, such as in Table 3, the easiest single value to obtain is the average. In Table 3, the average is $r_{CV} = 0.86$ with standard deviation 0.09. For

$$r_{CV} = 0.86$$

$$\rho_{CV} = (0.86)(0.92) = 0.791$$

$$\rho_{VC} = (0.86)(0.96) = 0.826$$

the stochastic generation of the sample sequence, with the same random numbers as in Figures 8 and 9, produced the sequence in Figure 10.

The inference to be drawn, in the comparison of Figures 8, 9, and 10, is that the average intercorrelation $r_{CV} = 0.86$ is an acceptable compromise, to be used instead of an assortment of values (Table 3). If, however, the non-lag correlation coefficient (r_{CV}) is reduced to a single value, then the lag correlation coefficients (ρ_{CV} , ρ_{VC}) also should be chosen for simplicity, thus the Whiton assumption (Eqs. 16).

8.2 Simulation of a Two-Station 48-h Sequence of Sky Covers

Suppose that the requirement is to simulate time changes of sky cover at two neighboring stations, like Boston and Bedford, Mass. in August when the diurnal effect is large. Each autocorrelation, per hour is,

$$\rho_{11} = \rho_{22} = 0.95$$

Suppose the intercorrelation is high, as it probably is:

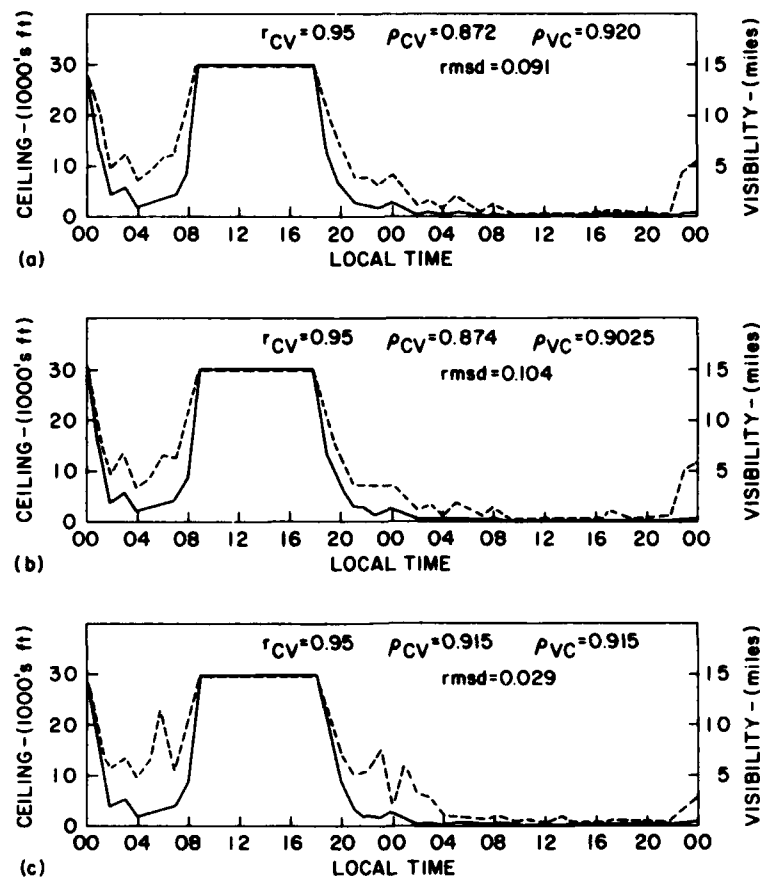


Figure 9. Similar to Figure 8 With Stronger Intercorrelations

$$r_{12} = 0.95$$

$$\rho_{12} = (0.95)(0.95) = 0.9025$$

$$\rho_{21} = (0.95)(0.95) = 0.9025$$

Somerville and Bean¹² have provided values for the two parameters (α , β) in the S-distribution of the cumulative probability, $F(x)$, of the sky cover $x = 0.0(0.1) 1.0$. The formula is

12. Somerville, P.N., and Bean, S.J. (1979) A New Model for Sky Cover, Scientific Report No. 5, Contract F19628-77-C-0080, AD 078368.

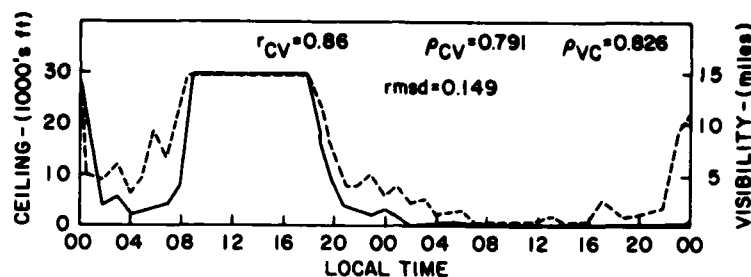


Figure 10. Similar to Figure 8 With Average Intercorrelations

$$F(x) = 1 - (1 - x^\alpha)^\beta \quad (37)$$

With the Somerville and Bean values used in the program (Appendix C), several 48-h sequences were obtained. Figure 11 depicts a close relationship of the two sky covers that remain broken to overcast during most of the 2-day period, yielding to scattered conditions during the most likely time of the evening. Figure 12, with another sequence of random numbers, depicts clear to scattered clouds in the first day, with the likely development of cloud in the afternoon. Figure 12 also depicts a 10-h interval when the sky cover differed markedly between stations. For the most part, however, the changes are concurrent.

Figures 11 and 12 should be compared with Figure 6. While Figures 11 and 12 give direct information on sky cover, the plot of Figure 6, on Normal Probability Paper, enables us to plot the climatic frequencies for comparison with the synoptic events. However, this is only a visual aid. For gaming purposes, the direct amount of sky cover may be desired as the product of the computer exercise.

9. DISCUSSION AND CONCLUSIONS

The simulation of a time sequence of changes of two interrelated variables is accomplished by using generating equations, basically Eq. (7). The coefficients (a's and b's) are determined in terms of correlation coefficients [Eqs. (12) and (13)]. The terms symbolized by r are the correlation coefficients between the variables without time lag, those symbolized by ρ are correlation

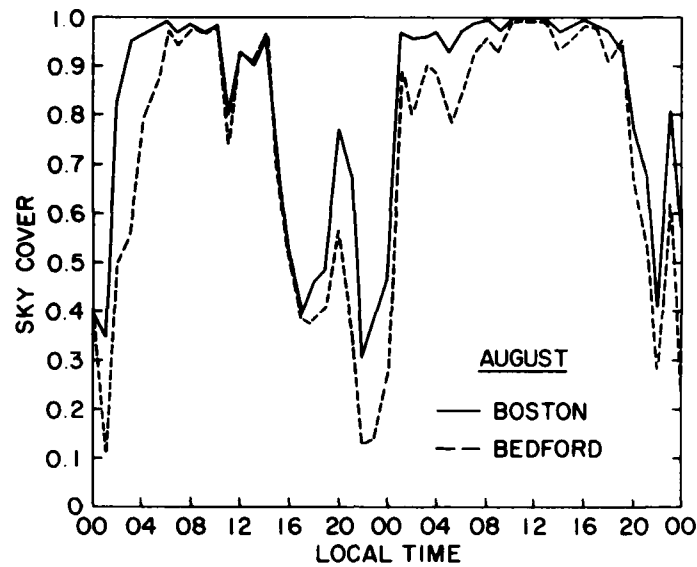


Figure 11. Stochastic Simulation of a 48-h Sequence of Sky Cover, When Autocorrelations are 0.95, $r_{12} = 0.95$, $\rho_{12} = \rho_{21} = 0.9025$. The curves are for Bedford, Mass. and Boston, Mass. August sky cover

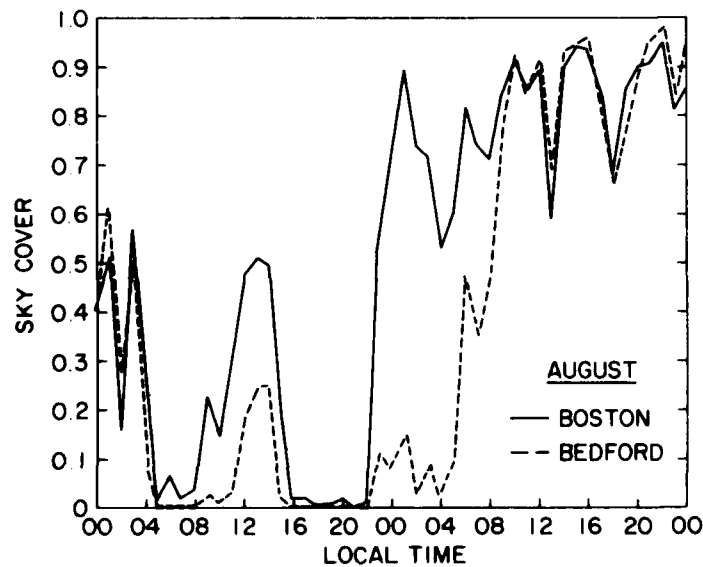


Figure 12. A Second Stochastic Sky-Cover Simulation. Conditions are the same as in Figure 11

coefficients with time lag. The term ρ_{cc} , for example, is the autocorrelation of ceiling with itself in time lag 1 h.

Unfortunately the correlation coefficients, lag correlations, or intercorrelations between variables, are not readily available, especially between END's. Special studies, using the hourly sequences of many years (10 or more) in each month need to be undertaken, to find such correlation coefficients. A measure of the intercorrelation, without time lag, between ceiling and visibility is obtainable from a RUSSWO.

Gringorten¹¹ published hour-to-hour autocorrelations of END's for ceilings, visibilities and other weather elements at Minneapolis, Minn. (Table 4). There were two estimates for each correlation; the first estimate was best for the duration of the weather element above a certain minimum, the second estimate was best for the duration of the weather element below a certain maximum. Generally speaking, the hour-to-hour correlation coefficient was higher for ceiling than for visibility. Other studies have indicated higher autocorrelation in winter than in summer although the figures for Minneapolis, Minn. do not support this conclusion. A "best" a priori approximation for hour-to-hour correlation is estimated to be 0.95.

Table 4. Estimates of Hour-to-Hour Autocorrelations of END's for Ceilings and Visibilities at Minneapolis, Minn., in Four Mid-Season Months

Element	Month			
	January	April	July	October
Ceiling	0.95	0.96	0.90	0.96
	0.95	0.97	0.92	0.97
Visibility	0.90	0.92	0.90	0.92
	0.94	0.95	0.95	0.95

Table 2 supports previous conclusions that correlation coefficients are lower in summer than in winter. Tables 2 and 3 reveal a regrettable dependence of the tetrachoric correlation coefficient on the measure of the ceiling and visibility. An overall average correlation coefficient, however, works well, to stochastically generate a sequence of values of two interrelated variables. Additionally, a time lapse simulation is well served by the Whiton estimates [Eqs. (16)] of lag correlations.

References

1. Bean, S.J., Somerville, P.N., and Heuser, M. (1979) Some Models for Ceiling, Scientific Report No. 7, Contract F19628-77-C-0080, AD A078033.
2. Somerville, P.N., Bean, S.J., and Falls, F. (1979) Some Models for Visibility, Scientific Report No. 3, Contract F19628-77-C-0080, AD A075490.
3. NBS (1964) Handbook of Mathematical Functions, Applied Mathematics Series, No. 55, Government Printing Office, Washington, D.C.
4. Kendall, M.G., and Buckland, W.R. (1971) A Dictionary of Statistical Terms, Hafner Publishing Co., New York.
5. Keilson, J., and Ross, H.F. (1979) Gaussian Markov Related Variates for Meteorological Planning, Final Report, Contract F19628-78-C-0158, AD A081382.
6. Gringorten, I.I. (1982) The Keilson-Ross Procedure for Estimating Climatic Probabilities of Duration of Weather Conditions, AFGL-TR-82-0116, AD A119860.
7. Whiton, R.C. (1982) Environmental Simulation Modeling, AWS/ETAC, Scott AFB, Ill.
8. NBS (1957) Tables of the Bivariate Normal Distribution Function, Applied Mathematics Series, No. 50, Government Printing Office, Washington, D.C.
9. Brooks, C.E.P., and Carruthers, N. (1953) Handbook of Statistical Methods in Meteorology, Her Majesty's Stationery Office, London.
10. Gringorten, I.I. (1979) Probability models of weather conditions occupying a line or an area, J. Appl. Meteorol. 18(No. 8):957-977.
11. Gringorten, I.I. (1966) A stochastic model of the frequency and duration of weather events, J. Appl. Meteorol. 5(No. 5):606-624.
12. Somerville, P.N., and Bean, S.J. (1979) A New Model for Sky Cover, Scientific Report No. 5, Contract F19628-77-C-0080, AD 078368.

Appendix A

Procedure to Generate a Simulated Sequence of END's in an O-U Process

Step 0. Begin with an arbitrary initial END, $[y(t)]$. Choose a value for the basic time interval (δt hours). Fix on the parameter: Relaxation Time (τ hours). Decide on the size, or the number of END-values in sequence (N).
Find

$$\rho = \exp(-\delta t / \tau)$$

Initialize $n = 0$.

Step 1. For $n = 0, N-1$

Increase n by 1. Use Subroutine (A) to find a random normal number,

$$\eta(t + \delta t).$$

Find

$$y(t + \delta t) = \rho \cdot y(t) + \sqrt{1 - \rho^2} \cdot \eta(t + \delta t)$$

If $n < N$, replace $y(t)$ with $y(t + \delta t)$ and repeat Step 1.

If $n = N$, the sequence is completed.

Subroutine A: For generating a random normal number.

(Note: This is only a simple suggested routine. There are others that are better.)

Step A. Begin with an arbitrary random number ($0 \leq x_0 \leq 1$) or the last number in the memory.

Set $J = 12$

$\sum x_j = 0.$

Step B. For $j = 0, J-1$

Increase j by 1

$$x_j = (x_{j-1} + \pi)^8 - \text{int}^\dagger (x_{j-1} + \pi)^8$$

Add x_j to $\sum x_j$

If $j < J$ replace x_{j-1} with x_j , and repeat Step B

For $j = J$, find $\eta(t + \delta t) = \sum x_j - 6$

RETURN to Step 1.

[†] int is the integral part of a number, for example, $\text{int}(12.375) = 12$

Appendix B

Procedure to Generate a Simulated Sequence of Two Interrelated END's (y_1, y_2) in a Markov Process

Step 0. Assign:

The non-lag correlation coefficient between y_1 and y_2 : r_{12}

The autocorrelation between $y_1(t)$ and $y_1(t + \delta t)$: ρ_{11}

The lag correlation coefficient between $y_1(t)$ and $y_2(t + \delta t)$: ρ_{12}

The lag correlation coefficient between $y_2(t)$ and $y_1(t + \delta t)$: ρ_{21}

The autocorrelation between $y_2(t)$ and $y_2(t + \delta t)$: ρ_{22}

Choose a value for the time interval: δt

Decide on the number of pairs (y_1, y_2) in sequence: N .

Find:

$$a_{11} = (\rho_{11} - r_{12} \rho_{21}) / (1 - r_{12}^2)$$

$$a_{12} = (\rho_{21} - r_{12} \rho_{11}) / (1 - r_{12}^2)$$

$$a_{21} = (\rho_{12} - r_{12} \rho_{22}) / (1 - r_{12}^2)$$

$$a_{22} = (\rho_{22} - r_{12} \rho_{12}) / (1 - r_{12}^2)$$

$$h_1 = \sqrt{1 - (\rho_{11}^2 + \rho_{12}^2 - 2r_{12}\rho_{11}\rho_{12}) / (1 - r_{12}^2)}$$

$$b_2 = \sqrt{1 - (\rho_{21}^2 + \rho_{22}^2 - 2r_{12}\rho_{21}\rho_{22})/(1 - r_{12}^2)}$$

$$h = [r_{12}(1 - a_{11}a_{22} - a_{12}a_{21}) - (a_{11}a_{21} + a_{12}a_{22})]/b_1b_2$$

Begin with an arbitrary pair of initial random END's: $y_1(0)$, $y_2(0)$.

Initialize $n = 0$

Step 1. For $n = 0, N-1$

Increase n by 1.

Use subroutine (A) to find a random normal number: $\eta_1(t + \delta t)$

Use subroutine (A) to find a random normal number: $\eta_2^1(t + \delta t)$.

Find:

$$\eta_2(t + \delta t) = h \cdot \eta_1(t + \delta t) + \sqrt{1 - h^2} \cdot \eta_2^1(t + \delta t)$$

Find:

$$y_1(t + \delta t) = a_{11} \cdot y_1(t) + a_{12} \cdot y_2(t) + b_1 \cdot \eta_1(t + \delta t)$$

$$y_2(t + \delta t) = a_{21} \cdot y_1(t) + a_{22} \cdot y_2(t) + b_2 \cdot \eta_2(t + \delta t)$$

If $n < N$, replace $y_1(t)$ with $y_1(t + \delta t)$

$y_2(t)$ with $y_2(t + \delta t)$ and repeat Step 1.

If $n = N$, the sequence is completed.

Subroutine A: For generating a random normal number (the same as in Appendix A).

Appendix C

Procedure to Generate a Simulated Sequence of Two Inter-related Weather Elements (X_1, X_2) in a Markov Process

Step 0. (Same as in Appendix B)

Step 1. For $n = 0, N-1$

Increase n by 1.

Find a random normal number: $\eta_1(t + \delta t)$

Find a random normal number: $\eta_2^1(t + \delta t)$.

Find:

$$\eta_2(t + \delta t) = h \cdot \eta_1(t + \delta t) + \sqrt{1 - h^2} \cdot \eta_2^1(t + \delta t)$$

$$y_1(t + \delta t) = a_{11} \cdot y_1(t) + a_{12} \cdot y_2(t) + b_1 \cdot \eta_1(t + \delta t)$$

$$y_2(t + \delta t) = a_{21} \cdot y_1(t) + a_{22} \cdot y_2(t) + b_2 \cdot \eta_2(t + \delta t)$$

Step 2. Use Subroutine (B) to find the probability (p_1) corresponding to $y_1(t + \delta t)$

Use Subroutine (B) to find the probability (p_2) corresponding to $y_2(t + \delta t)$

Find the hour of the day (L):

$$L = 24 \left\{ \text{fra} \left(\frac{n}{24} \right) \right\}^{\dagger}$$

Find the hourly group (g):

$$g = \text{int}(L/3)^{\dagger\dagger}$$

Use Subroutine (C) to find $X_1(t + \delta t)$ corresponding to p_1

Use Subroutine (D) to find $X_2(t + \delta t)$ corresponding to p_2

If $n < N$, replace $y_1(t)$ with $y_1(t + \delta t)$

replace $y_2(t)$ with $y_2(t + \delta t)$ and repeat Steps 1, 2.

If $n = N$, the sequence is completed.

Subroutine A: For generating a random normal number (the same as in Appendix A).

Subroutine B: For transforming an END (y) into probability, $p = P(\leq y)$

Assign:

$$c_1 = 0.196\ 854$$

$$c_2 = 0.115\ 194$$

$$c_3 = 0.000\ 344$$

$$c_4 = 0.019\ 527$$

Set $l = 1$, $m = -1$ for $y \geq 0$

Set $l = 0$, $m = 1$ for $y < 0$.

$$\text{Find } p = l + m \left\{ 2(1 + c_1|y| + c_2y^2 + c_3|y^3| + c_4y^4)^4 \right\}^{-1}$$

RETURN.

Subroutine C: For ceiling height (C), given $P (\leq C)$

Find, from computer memory, the values a, b, c for the hourly group (g).

Where $p = P(\leq C)$

Find

$$C = c \left[(1 - p)^{1/b} - 1 \right]^{1/a}$$

RETURN or alternative.

[†]fra is the fractional part of a number, for example, $\text{fra}(12.375) = 0.375$

^{††}int is the integral part of a number, for example, $\text{int}(12.375) = 12$

Subroutine C: For cloud cover (x), given $P(\leq x)$

Find, from computer memory, the values α , β for the hourly group (g)

Where $p = P(\leq x)$

Find

$$x = \left[1 - (1 - p)^{1/\beta} \right]^{1/\alpha}$$

RETURN.

Subroutine D: For visibility (V) given $P(\leq V)$

Find, from computer memory, the values α , β for the hourly group (g)

Where $p = P(\leq V)$

Find

$$V = \left[\frac{1}{\alpha} \ln \left(\frac{1}{1 - p} \right) \right]^{1/\beta}$$

RETURN, or alternative.

Subroutine D: For cloud cover (same as for Subroutine C for cloud cover).

END

FILMED

11-83

DTIC